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ORIGINAL ARTICLE

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Energy loss due to radiation in postmortem cooling

Part A: Quantitative estimation of radiation using the Stefan-Boltzmann law

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Abstract Conduction and convection are assumed to account for most of the energy loss from the dead body to the (cooler) environment. There are no quantitative estimations in the literature for the contribution of radiation to heat loss. The aim of the present paper was to estimate the radiation energy loss in postmortem cooling. The Stefan-Boltzmann law is used and combined with a single-exponential model for the cooling process of the skin derived from experimental data of Lyle and Cleveland (1956). The influence of various factors (e.g. skin temperature, environmental temperature, body mass and body height) on the amount of radiation emitted was investigated. The radiation energy is quantitatively described as a function of time. The radiation energy loss ranged from approximately 200 kJ in small (165 cm) and lean (50 kg) bodies at room temperature (20 °C) to approximately 600 kJ in tall (185 cm) and over-weight (100 kg) bodies at outdoor temperature (5 °C) in the first hour postmortem.

Key words Postmortem cooling · Radiation energy loss · Stefan-Boltzmann law · Single exponential model · Skin temperature

Introduction

Temperature-based methods play an important role in the estimation of time since death in the early postmortem period. Of the main mechanisms of heat transfer, conduction and convection are commonly considered to be responsible for the heat transfer from a dead body to the environment if the environmental temperature lies below the body temperature [6]. Energy loss by radiation is, according to most authors, assumed to be very small or even negligible. Nevertheless some authors [10] deal with the variability of energy loss by radiation depending on the body site, others [8] consider radiative and convective energy losses to be about equal, without quantifying the amount of radiation. The temperature-based methods [3, 4, 11] mathematically describing cooling processes of corpses investigated by experiment, implicitly include all mechanisms of heat tranfer. On the contrary, the methods of thermodynamical modelling [7, 9, 14] are commonly based on the differential equation of conductive energy tranfer and it is essential to know the contribution of the different mechanisms (especially radiation) to the overall heat transfer. The present paper therefore has the following aims:

1. Calculation of the energy loss due to radiation with reference to different conditions (e.g. environmental temperature, body weight, body height).
2. Presentation of the amounts of radiation energy as a function of time and temperature.

Dedicated to Professor M. Staak on the occasion of his 65th birthday

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Materials and methods

The cooling process of the skin

Heat-radiation is, by nature, emitted only from the outer surfaces of warm objects – in the case of human bodies from the skin. In analogy to the mathematical descriptions in the forensic literature, cooling of the exposed skin can be mathematically described by an exponential approach of the following formula:

$$\frac{T_S - T_E}{T_{SO} - T_E} = e^{-Z' t} \quad (1)$$

where T_{S0} represents the skin temperature at time t = 0, T_S the skin temperature at time t > 0 and T_E the environmental temperature (assumed as constant). Equation (1) can be equivalently expressed by:

$$T_S(t) = T_S = (T_{S0} - T_E)\, e^{-Z' t} + T_E \quad (1a)$$

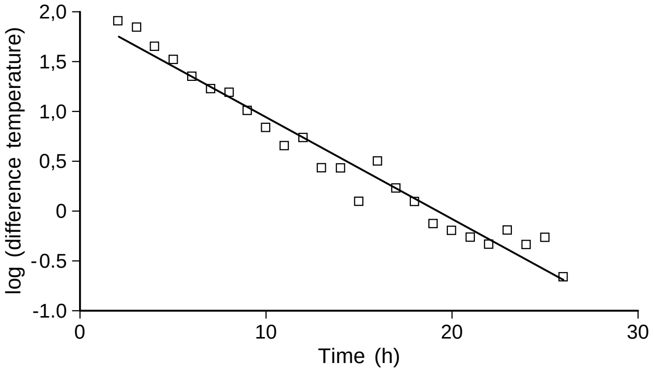

Fig. 1 Loglinear regression analysis of the temperature difference data by Lyle and Cleveland (1956) for exposed skin (forehead)

Substituting $\alpha = T_{S0} - T_E$ and $\beta = T_E$ leads to the more compact form:

$$T_S(t) = \alpha\, e^{-Z' t} + \beta \qquad (1b)$$

Of the various forensic studies dealing with the cooling of skin [7, 10, 13, 15], only the study of Lyle and Cleveland [10] provides cooling curves for the exposed human skin (forehead). Their measurements were taken under standardized conditions in 56 corpses at constant environmental temperature. They present overall data for the temperature difference between skin and surroundings ranging from 1 to 12 ° Fahrenheit. We applied the natural logarithm on the pure numbers of the temperature data after conversion from °F to °C and carried out a highly significant linear regression analysis (Fig. 1), deriving a gradient Z′, of which the absolute value is multiplied with the unit h^{-1}:

$$Z' = 0.1017\ h^{-1}$$

Although the value for Z′ is derived from data dealing with comparatively small temperature differences (of about 7 °C), it is transferred to cooling processes with greater temperature differences for the present estimations.

According to the thermographic studies of Newitt and Green [13] and to the data supplied by Gagge and Gonzalez [2], the mean temperature of the skin T_{S0} is assumed to be equal to 33 °C (306 °K).

The black body and the Stefan-Boltzmann law

The concept of the black body was introduced in thermodynamics to solve the problem of energy transfer by radiation. It is a hypothetical body which perfectly absorbs radiation of arbitrary frequency and direction. The radiation power P_{BV} of the black body in a vacuum at a given temperature T (in °K) and with a radiating surface area A_R (in m^2) is governed by the law of Stefan and Boltzmann:

$$P_{BV}(T) = \sigma A_R T^4 \qquad (2)$$

with the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/(m^2 °K^4).

The radiation power P_{RV} of a real body can be determined by correcting (2) with a factor ε called emissivity:

$$P_{RV}(T_S) = \varepsilon \sigma A_R T_S^4 \qquad (2a)$$

The emissivity is a function of several variables, especially temperature. In the temperature ranges occurring in postmortem cooling, the human skin is often called a “black body radiator” [12]. According to Brück [1], the emissivity of the skin can be approximated by ε = 1. According to other investigators, the skin has an emissivity of ε = 0.98 [2] or ε = 0.95 [16]. For our calculations we assume the median of the values:

$$\varepsilon = 0.98$$

The body surface area A_D is calculated according to the formula of Dubois [2]:

$$A_D = 0.202\ m^{0.425} h^{0.725}$$

where m represents the pure number of the body weight in kg, h the pure number of the body height in m and A_D the surface area in m^2.

For the following calculations a position of the body during the process of cooling corresponding to the standard conditions (naked body, lying extended on the back, on a thermally indifferent ground) of Henßge [3] is assumed. In this position a radiating area A_R has to be defined, since the skin of the back as well as the skin of the medial sides of the upper and lower extremities do not contribute to the radiation emitted. According to the correction factors given by Gagge and Gonzalez [2], A_R can be determined by reducing the Dubois surface area A_D:

$$A_R = 0.5\ A_D$$

Calculation of the radiation energy loss in the cooling process

The formulae (2) and (2 a) describe the radiation power of bodies in a vacuum at an environmental temperature T_E = 0° K without taking into account any environmental influence. In reality, a term for the absorption of the radiation from the environment has to be subtracted. According to the law of Kirchhoff, this term can be written, as follows:

$$P_{RE}(T_E) = \varepsilon \sigma A_R T_E^4 \qquad (2b)$$

under the precondition of a constant environmental temperature T_E. The radiating power of a human body P_R of the skin temperature T_S in an environment (air and surface of surrounding walls) of temperature T_E is then:

$$P_R(T_S, T_E) = P_{RV}(T_S) - P_{RE}(T_E) \qquad (2c)$$

Inserting formulae (2 a) and (2 b) leads to:

$$P_R(T_S, T_E) = \varepsilon \sigma A_R (T_S^4 - T_E^4) \qquad (2d)$$

If P_R (t) denotes the radiation power of the skin at time t in the process of cooling of the skin of a corpse with the radiating surface area A_R under standard conditions [3, 6] from temperature T_{S0} at time t = 0 to temperature T_S at time t > 0, the energy E_R (t) due to radiation is:

$$E_R(t) = \int_0^t P_R(t')\, dt' \qquad (3)$$

where t’ denotes the substitute for the time variable t in the integral expression.

As presented in detail in the appendix, it is possible to obtain an analytical expression for the amount of energy loss E_R (t):

$$E_R(t) = f(t, T_{S0}, T_E, A_R, \varepsilon, Z') \qquad (4)$$

The energy difference (energy loss) ΔE_R, emitted by the skin during a time interval [t –Δ t, t], can be expressed by:

$$\Delta E_R(t) = E_R(t) - E_R(t - \Delta t) \qquad (5)$$

An algorithm for computing equation (4) has been developed and the program (programming language: Turbo Pascal 7.0) is implemented on a personal computer (Pentium processor, 166 Mhz).

Results

Taking up the aims mentioned in the introduction we obtain the following estimations:

1. Tables 1 and 2 present the calculations for two different body sizes, 165 cm and 185 cm, and different body weights, 65 kg ± 15 kg and 85 kg ± 15 kg. Table 1 presents the calculations for an environmental temperature of 20 °C (e.g. room temperature). Table 2 presents

Table 1 Estimated amounts of radiation energy loss ΔE_R in time intervals $\Delta t = 1$ h at an environmental temperature of $T_E = 20\,°C$

Time since death t in hours	Skin temperature T_S in °C	165 cm 50 kg 1.53 m²	165 cm 65 kg 1.71 m²	165 cm 80 kg 1.87 m²	185 cm 70 kg 1.91 m²	185 cm 85 kg 2.08 m²	185 cm 100 kg 2.23 m²
0	33.0	0	0	0	0	0	0
1	31.7	203	227	248	253	276	295
2	30.6	182	203	222	227	247	265
3	29.6	164	183	200	204	222	238
4	28.7	147	164	180	183	200	214
5	27.8	132	148	162	165	180	193
6	27.1	119	133	145	148	162	173
7	26.4	107	120	131	134	145	156
8	25.7	96	108	118	120	131	140
9	25.2	87	97	106	108	118	126
10	24.7	78	87	96	98	106	114
11	24.2	70	79	86	88	96	103
12	23.8	63	71	78	79	86	93
13	23.5	57	64	70	71	78	83
14	23.1	52	58	63	64	70	75
15	22.8	47	52	57	58	63	68
16	22.6	42	47	51	52	57	61
17	22.3	38	42	46	47	51	55
18	22.1	34	38	42	43	46	50
19	21.9	31	34	38	38	42	45
20	21.7	28	31	34	35	38	41
21	21.5	25	28	31	31	34	37
22	21.4	23	25	28	28	31	33
23	21.2	20	23	25	26	28	30
24	21.1	18	21	23	23	25	27
25	21.0	17	19	20	21	23	24
26	20.9	15	17	18	19	20	22
27	20.8	14	15	17	17	18	20
28	20.8	12	14	15	15	17	18
29	20.7	11	12	14	14	15	16
30	20.6	10	11	12	12	14	15
31	20.6	9	10	11	11	12	13
32	20.5	8	9	10	10	11	12
33	20.5	7	8	9	9	10	11
34	20.4	7	7	8	8	9	10
35	20.4	6	7	7	7	8	9
36	20.3	5	6	7	7	7	8
37	20.3	5	5	6	6	7	7
38	20.3	4	5	5	6	6	6
39	20.2	4	4	5	5	5	6
40	20.2	4	4	4	5	5	5
41	20.2	3	4	4	4	4	5
42	20.2	3	3	4	4	4	4
43	20.2	3	3	3	3	4	4
44	20.1	2	3	3	3	3	4
45	20.1	2	2	3	3	3	3
46	20.1	2	2	2	2	3	3
47	20.1	2	2	2	2	2	3
48	20.1	2	2	2	2	2	2
49	20.0	1	2	2	2	2	2
50	20.0	1	1	2	2	2	2

the calculations for an environmental temperature of 5 °C (e.g. outdoor temerature). The first column gives the time since death t in hours, the second column the corresponding skin temperature T_S in °C. The remaining columns contain the amounts of radiation energy ΔE_R in kJ related to time intervals $\Delta t = 1$ h.

2. Figures 2–4 graphically present the time-dependent course of the amounts of the radiation energy differ-

Table 2 Estimated amounts of radiation energy loss ΔE_R in time intervals $\Delta t = 1$ h at an environmental temperature of $T_E = 5\,°C$

Time since death t in hours	Skin temperature T_S in °C	165 cm 50 kg 1.53 m²	165 cm 65 kg 1.71 m²	165 cm 80 kg 1.87 m²	185 cm 70 kg 1.91 m²	185 cm 85 kg 2.08 m²	185 cm 100 kg 2.23 m²
0	33.0	0	0	0	0	0	0
1	30.3	404	451	493	504	549	588
2	27.8	360	402	439	449	489	524
3	25.6	321	359	392	401	436	468
4	23.6	287	320	350	358	390	418
5	21.8	256	287	313	320	349	374
6	20.2	229	256	280	286	312	334
7	18.7	206	230	251	257	279	300
8	17.4	184	206	225	230	251	269
9	16.2	165	185	202	206	225	241
10	15.1	148	166	181	185	202	216
11	14.1	133	149	163	166	181	194
12	13.2	120	134	147	150	163	175
13	12.4	108	120	132	135	147	157
14	11.7	97	108	119	121	132	141
15	11.0	87	98	107	109	119	127
16	10.5	79	88	96	98	107	115
17	10.0	71	79	86	88	96	103
18	9.5	64	71	78	80	87	93
19	9.0	57	64	70	72	78	84
20	8.7	52	58	63	65	70	75
21	8.3	47	52	57	58	63	68
22	8.0	42	47	51	53	57	61
23	7.7	38	42	46	47	52	55
24	7.4	34	38	42	43	47	50
25	7.2	31	35	38	39	42	45
26	7.0	28	31	34	35	38	41
27	6.8	25	28	31	31	34	37
28	6.6	23	25	28	28	31	33
29	6.5	20	23	25	26	28	30
30	6.3	18	21	23	23	25	27
31	6.2	17	19	20	21	23	24
32	6.1	15	17	18	19	20	22
33	6.0	14	15	17	17	18	20
34	5.9	12	14	15	15	17	18
35	5.8	11	12	14	14	15	16
36	5.7	10	11	12	12	14	15
37	5.7	9	10	11	11	12	13
38	5.6	8	9	10	10	11	12
39	5.5	7	8	9	9	10	11
40	5.5	7	7	8	8	9	10
41	5.4	6	7	7	7	8	9
42	5.4	5	6	7	7	7	8
43	5.4	5	5	6	6	7	7
44	5.3	4	5	5	6	6	6
45	5.3	4	4	5	5	5	6
46	5.3	4	4	4	5	5	5
47	5.2	3	4	4	4	4	5
48	5.2	3	3	4	4	4	4
49	5.2	3	3	3	3	4	4
50	5.2	2	3	3	3	3	3

ence ΔE_R (related to the time intervals Δt given on the abscissa), of the total amount of radiation energy E_R (emitted in the time interval from 0 to t) as well as of the skin temperature T_S. Figure 2 describes the short-term course up to 5 h postmortem with time intervals $\Delta t = 15$ min, Fig. 3 the medium-term course up to 40 h postmortem with time intervals $\Delta t = 2$ h and Fig. 4 the long-term course up to 80 h postmortem with time in-

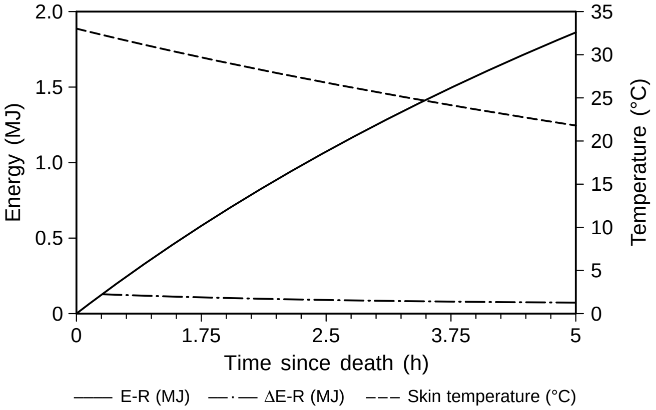

Fig. 2 Radiation energy loss Δ E_R in time intervals Δ t = 1/4 h, cumulative radiation energy loss E_R and skin temperature T_S up to 5 h postmortem (body size: 175 cm, body weight: 75 kg, environmental temperature: 5 °C)

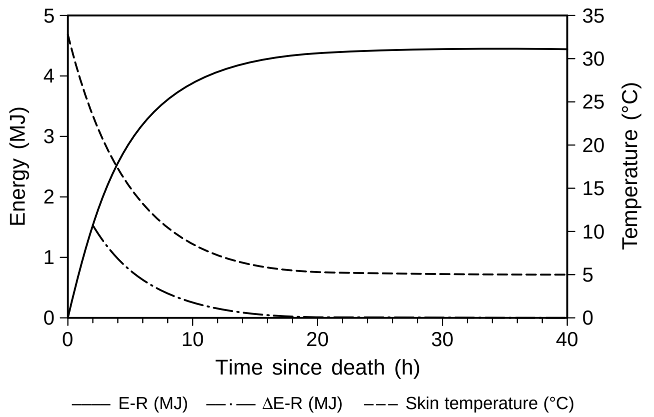

Fig. 3 Radiation energy loss Δ E_R in time intervals Δ t = 2 h, cumulative radiation energy loss E_R and skin temperature T_S up to 40 h postmortem (body size: 175 cm, body weight: 75 kg, environmental temperature: 5 °C)

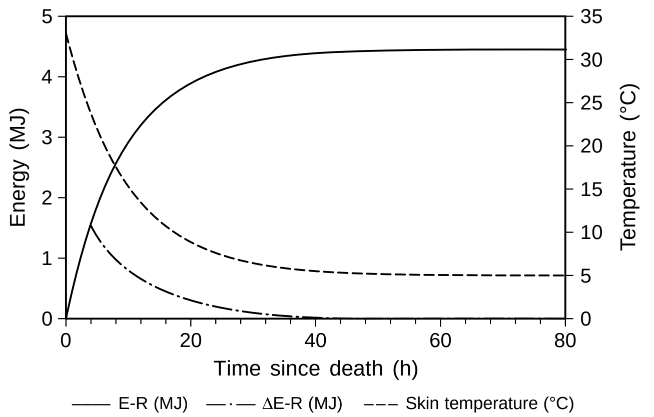

Fig. 4 Radiation energy loss Δ E_R in time intervals Δ t = 4 h, cumulative radiation energy loss E_R and skin temperature T_S up to 80 h postmortem (body size: 175 cm, body weight: 75 kg, environmental temperature: 5 °C)

tervals Δ t = 4 h. The values were calculated for a middle-sized (175 cm) and medium-weight (75 kg) individual (A_D = 1.9 m^2) at an environmental temperature of 5 °C.

Discussion

The knowledge of the exact time of death is of great importance in medico-legal practice. In the early postmortem period, temperature-based methods, e.g. the method of Henßge [3, 4], contribute most to the estimation of the time since death. Many authors have tried to combine the physical laws of heat transfer and the experimentally investigated cooling processes in corpses. Sellier [14] solved the differential equation of heat transfer by approximating the human body to a cylinder of infinite length. Joseph and Schickele [8] also referred to the cylindrical model and analysed the heat flow under varying insulation conditions. Marshall and Hoare [11] and Henßge [3] were able to achieve a very good match of a double exponential model to the experimental data obtained. It is a great contribution of Henßge [4] to have determined a variety of coefficients accounting for different clothing and environmental conditions. Another very different method for estimating the time since death by temperature decrease is the thermodynamic modelling as presented by Kuehn et al. [9] and Hiraiwa et al. [7].

The main mechanisms of heat transfer are conduction as energy transfer from one particle to another, convection as heat transfer by the particles themselves and radiation as heat transfer by electromagnetic waves. Conduction and convection are, according to a widely accepted opinion, assumed to account for most of the heat loss in the cooling of a corpse [6]. Heat loss due to radiation is mostly considered to be negligibly small. This is concluded indirectly from the fact that insulation of a body by e.g. clothing leads to a slowing down of the cooling process [6]. But the absorption of the radiation emitted from the skin by the layers of clothing has also to be taken into account and may very well explain a slowing down of the cooling process. Lyle and Cleveland [10] speak of a considerable variation of the amount of radiation depending on the site of the body but do not provide any quantitative rates for the radiation. Joseph and Schickele [8] consider the amount of radiative and convective heat loss to be about equal without quantifying either of them.

Most mathematical formulations describing the process of postmortem cooling are based on the differential equation of heat transfer, genuinely comprising only the conductive transfer. The mathematical description of the convective heat transfer would additionally need further (fluid and gas) dynamic considerations leading to nonlinearity of the equation. The radiation heat transfer is described by the law of Stefan and Boltzmann. A comprehensive formulation would require the knowledge of the portions of heat loss due to the above mechanisms and at the same time lead to a very complex formula which is difficult to apply in practice. The temperature-based methods [3, 4, 11] avoid these difficulties by determining overall coefficients summarizing all the mechanisms of heat loss. For thermodynamical modelling [7, 9, 14] it is essential to differentiate between the mechanisms of heat transfer as they follow different laws.

From the calculations presented it follows that:

1. The energy emitted by the skin in the process of postmortem cooling amounts to several 100 kJ (in the present estimations: 200–600 kJ) in the first hour postmortem. It still has values of about 100 kJ per hour in the later process of cooling (in the present estimations: up to 7–18 h postmortem) depending on the decrease of skin temperature.
2. The time-dependent course of the radiation energy ΔE_R emitted in time intervals Δt is, in general, a linear combination of exponential functions as suggested by formula (4) and (A 5). The short-term curve has an almost linear character, as expected from a Taylor series expansion of order one of formula (4) or (A 5).

Our results may explain why the mathematical method of Henßge [3] provides more accurate results than direct thermodynamical approaches [5]. The fitting parameters of the mathematical approach by Henßge [3, 4] implicitly take into account the non-conductive effects of thermal energy loss, especially radiation, while the thermodynamic models [7, 9, 14] are exclusively based on the differential equation of heat conduction. The estimations presented in the tables may serve as a basis for evaluating the necessity of a separate radiation approach in thermodynamical modelling in different cases and for different conditions. The estimations presented are based on several general assumptions and valid only for standard conditions. They are intended as a basis for further analyses of radiation under non-standard conditions. Experiments for the determination of the skin cooling curve and validation experiments for the radiation-time curve are planned.

In part B of the paper the energy balance with respect to radiation energy loss will be presented, comparing the radiation energy loss to the total thermal energy loss, thereby giving a rough estimation of a lower bound for metabolic energy production due to supravital activity.

Appendix

Inserting (1 b) into (2 d) and assuming T_E = const. for computing P_R (T_S, T_E) over [0, t], we obtain through (3):

$$E_R(t) = \varepsilon \sigma A_R \int_0^t \left([\alpha e^{-Z' t'} + \beta]^4 - \beta^4 \right) dt' \qquad (A1)$$

Applying the binomial formula and using the linearity of the integral leads to:

$$E_R(t) = \varepsilon \sigma A_R \left(\sum_{j=0}^{4} \binom{4}{j} \alpha^{4-j} \beta^j \left[\int_0^t e^{-(4-j) Z' t'} dt' \right] - \int_0^t \beta^4 dt' \right) \qquad (A2)$$

The integral expression on the right side of (A 2) is abbreviated by:

$$I_J(t) := \int_0^t e^{-(4-j) Z' t'} dt' \quad \forall j = 0, \ldots, 4 \qquad (A3)$$

The integrals (A 3) are solved by:

$$I_J(t) = \begin{cases} t & j = 4 \\ ((4-j) Z')^{-1} \; (1 - e^{-(4-j) Z' t}) & \text{else} \end{cases} \qquad (A4)$$

Equation (A 4) is inserted in (A 2):

$$E_R(t) = \varepsilon \sigma A_R \sum_{j=0}^{4} \left[\binom{4}{j} \alpha^{4-j} \beta^j \frac{1 - e^{-(4-j) Z' t'}}{(4-j) Z'} \right] - \beta^4 t \qquad (A5)$$

(A5) leads to the desired formula of the form (4).

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